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RESEARCH MEMORANDUM

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Abstract

This paper presents a management oriented reorder technique based on a continuous (s,Q) inventory model, which does not require any marginal cost information.
The procedure - which is based on aggregate objectives and constraints - results in a point located on an optimal three dimensional response surface, showing optimal relationships among aggregate customer service, workload (total number of replenishments) and investment. By comparing our coordinated procedure with the classical EOQ-model, we will show the superiority of the simultaneous approach.
Especially demand which is highly erratic in nature will leave us with significant cost differences between the two approaches.

Keywords:

Inventory, aggregate analysis, optimal policy surface, workload, investment budget, service constraint.

1. Introduction

In the authors' opinion a serious gap exists between the theoretical solutions on the one hand, and the real world problems on the other (see also Gardner and Dannenbring [4]; Silver and Peterson [12]).
We will try to bridge this gap in several ways.
First we base our analysis on aggregate objectives and constraints. By aggregating we overcome the most serious inconveniences of classical single-item models. In practice top management is responsible for thousands of items and is primarily concerned with aggregate variables. In addition one has to operate under service constraints stated by the marketing department and investment constraints fixed by the financial department. In old - established literature (Brown [2]; Plossl and Wight [8]) one tackles this problem by solving a cost minimalization problem, with a service resultant and neglecting any financial restrictions.
In the second place we will try to avoid the prolem of a correct cost parameter determination. In this context the carrying charge appears to be the most complicating factor, as its value should not be based on theoretical aspects, such as return on investment for example.
Instead it ought to be an <u>instrument variable</u> . According to Brown [2]: "There is no "correct" value to use, other than the value that results in what management wants" (see also Silver and Peterson [12]). Over and above that the cost determination is heavily dependent on the accounting system in use, it is not surprising that cost parameters can not uniquely be established. Furthermore the cost accounting output produces average figures in most cases, while only marginal costs contain relevant information. (see also Selen and Wood [11]; Schonberger and Schniederjans [10]).
In this research, inventory decisions will be considered as a policy tradeoff between aggregate customer service workload and investment, without any need to determine marginal cost figures.

2. A two-dimensional aggregate inventory analysis

Besides the complexity of an explicit determination (see also the introduction) of the carrying charge (r) and marginal fixed order cost (A),

it's even unwise to minimize total cost on an individual basis. Most practioners are primarily concerned with aggregate inventory control, taking into account aggregate objectives and constraints such as workload and investment.
The traditional single-item analysis provides insufficient insights into existing interdependencies between items under control and conflicting intern business objectives. To gain more insight into the results on exchanges curves as presented by Silver and Peterson [12], we will next formally prove the optimality relation between both decision variables: total cycle stock investment (y) and workload (N), using a standard Lagrange optimization technique. When an EOQ-strategy is used for each item, one can derive every individual EOQ_i-value (i=1,...n) from the resulting workload associated with the total cycle stock investment fixed by top-management.
Summarizing the above reasoning the following steps are in order:

- the financial department fixes the total investment in cycle stocks ($y=\bar{y}$, a fixed value);
- given this budget it's possible to determine the optimal workload ($N=N^*$);
- given this information it is possible to derive the (A/r)-value and the individual EOQ_i- values (i=1,...,n).

We now derive to above discussed optimality relation.

The objective function subject to the investment constraint is as follows:

$$\min_{q_i} N = \sum_{i=1}^{n} \frac{d_i}{q_i} \tag{2.1}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} \frac{q_i v_i}{2} = y \tag{2.2}$$

where

N = total annual replenishments (= workload).

d_i = annual sales in units for item i

q_i = order quantity in units for item i

y = total annual cycle stock investment

v_i = unit value of item i expressed in guilders per unit.

The Lagrangean function becomes:

$$L\ (q_1,\dots,q_n,\lambda_y) = \sum_{i=1}^{n} \frac{d_i}{q_i} + \lambda_y \left[\sum_{i=1}^{n} \frac{q_i v_i}{2} - y\right], \tag{2.3}$$

where λ_y = Lagrangean multiplier with respect to the investment budget. Differentiating with respect to q_i, λ_y $(i=1,\dots n)$ we obtain the following first order conditions:

$$\frac{\partial L}{\partial q_i} = \frac{-d_i}{q_i^2} + \frac{\lambda_y . v_i}{2} = 0,\ (i=1,\dots n); \tag{2.4}$$

$$\frac{\partial L}{\partial \lambda_y} = \sum_{i=1}^{n} \frac{q_i v_i}{2} - y = 0. \tag{2.5}$$

After some algebraic manipulations we obtain the following results:

$$\lambda_y = \frac{N}{y} \tag{2.6}$$

and

$$q_i = \sqrt{\frac{2\ d_i}{\lambda_y\ v_i}},\ (i=1,\dots,n), \tag{2.7}$$

which after substitution leads to:

$$q_i = \sqrt{\frac{2.d_i.y}{N.v_i}},\ (i=1,\dots,n). \tag{2.8}$$

Therefore equation (2.1) can be rewritten:

$$N = \sum_{i=1}^{n} \frac{d_i}{\sqrt{\frac{2.d_i.y}{N.v_i}}} = \sum_{i=1}^{n} \sqrt{\frac{N.d_i.v_i}{2.y}} = \sqrt{\frac{N}{y}} \cdot \frac{1}{\sqrt{2}} \sum_{i=1}^{n} \sqrt{d_i.v_i} \tag{2.9}$$

or

$$\sqrt{N} \;.\; \sqrt{y} = \tfrac{1}{2}\,\sqrt{2}.\; \left[\sum_{i=1}^{n} \sqrt{d_i . v_i}\right], \tag{2.10}$$

which is equivalent with

$$N \;.\; y = \tfrac{1}{2} \;.\; \left[\sum_{i=1}^{n} \sqrt{d_i . v_i}\right]^2 . \tag{2.11}$$

The above formula (2.11) is an hyperbola.

Using an EQO-stategy for each item we will obtain two more relations with which we can demonstrate the equivalence with the above Lagrangean result.

$$y = \sum_{i=1}^{n} \frac{q_i v_i}{2} = \sum_{i=1}^{n} \frac{\sqrt{\dfrac{2A.d_i}{v_i.r}} \;.\; v_i}{2}$$

$$= \sum_{i=1}^{n} \sqrt{\frac{A}{r}} \;.\; \frac{1}{\sqrt{2}} \;.\; \sqrt{d_i.v_i}$$

$$= \sqrt{\frac{A}{r}} \;.\; \frac{1}{\sqrt{2}} \;.\; \sum_{i=1}^{n} \sqrt{d_i.v_i} \tag{2.12}$$

$$N = \sum_{i=1}^{n} \frac{d_i}{q_i} = \sum_{i=1}^{n} \frac{d_i}{\sqrt{\dfrac{2A.d_i}{v_i.r}}}$$

$$= \sum_{i=1}^{n} \sqrt{\frac{r}{A}} \;.\; \frac{1}{\sqrt{2}} \;.\; \sqrt{d_i.v_i}$$

$$= \sqrt{\frac{r}{A}} \;.\; \frac{1}{\sqrt{2}} \;.\; \sum_{i=1}^{n} \sqrt{d_i.v_i} \tag{2.13}$$

So we can verify:

$$y \cdot N = \tfrac{1}{2} \cdot \left[\sum_{i=1}^{n} \sqrt{d_i \cdot v_i} \right]^2 . \tag{2.14}$$

At the same time we obtain:

$$\frac{y}{N} = \frac{A}{r} \quad , \tag{2.15}$$

which is equivalent to formula (5.29) and (5.30) from Silver and Peterson [12].

Note the equivalence between formula (2.6) and (2.15):

$$\lambda_y = \frac{N}{y} = \frac{r}{A} \quad . \tag{2.16}$$

From (2.16) it's clear that a (r/A)-value is implied by fixing a multiplier value. A graphical representation of formula (2.14) is to be found in figure (2.1):

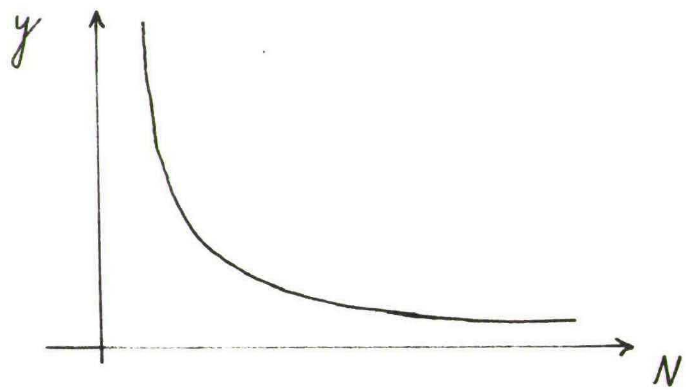

Figure 2.1. Example of the exchange curve

From relation (2.15) it's easily seen that any point on the hyperbolic curve implies an unique value of $\frac{A}{r}$.

Alternative approaches can be found in Plossl and Wight [8], Eaton [3], Prichard and Eagle [9], Groff and Muth [5], Hadley and Whitin [6].

Silver and Peterson [12] give an excellent overview of the exchange curve concept, of which a short exposition will be given next.

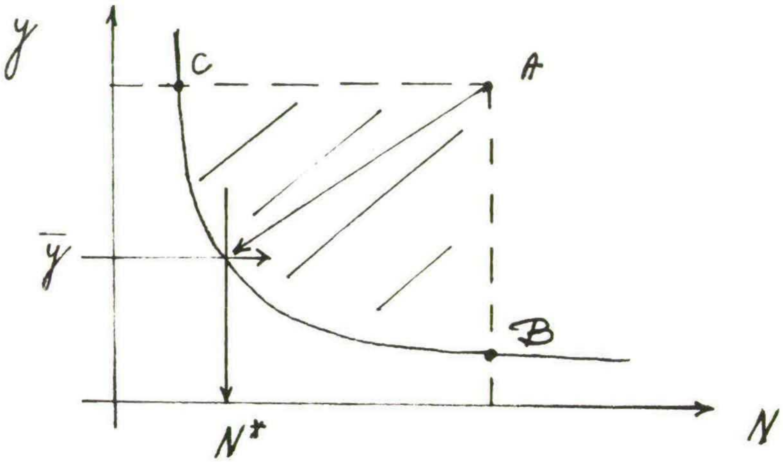

Figure 2.2. Aggregate performance of different $\frac{A}{r}$ values.

Notation used:

A: estimated current operating point in terms of total number of replenishments per year (N) and total average cycle stock (y).

B: point achieved by cutting total average cycle stock (y) at the same total annual number of replenishments (N).

C: point achieved by cutting the total number of replenishments per year (N) at the same total average cycle stock (y).

D: optimal point achieved by the total average cycle stock ($\bar{y}$) fixed by the financial department.

Given the feasible policy alternatives, management should try to improve their current policy by moving closer in a left downward direction to the exchange curve with regard to the current operating point A.
Quoting Silver and Peterson [12]:
"From a top management perspective this is far more appealing than the fact that the EOQ minimizes total costs on an individual item basis. (Particularly when the individual cost factors are so difficult to estimate)".

The solution procedure is presented in short to clarify the general concept:

1. Management fixes the total annual cycle stock investment ($\bar{y}$).
2. Via equation (2.11) the minimal number of replenishments (N^*) can be determined because there exists an optimal relationship between aggregate average cycle stock investment and workload, when demand is deterministic.
   The point on the exchange curve which is found in this manner, determines implicitly an unique A/r - value, which can be obtained with formula (2.15).
3. Via equation (2.8) the order quantities (q_i,i=1,...,n) can be determined without explicitly having to estimate the marginal order cost (A) and carrying charge (r).
4. An improvement on the actual inventory performance can be obtained by moving in figure 2.2 from the estimate current operating point A towards the optimal point D on the so called exchange curve.

Implicitly we have assumed a direct splitting up of the total available inventory investment budget (Y) into a total cycle stock investment (y) and a total safety stock budget. The latter being equal to:

$$Y - y = \sum_{i=1}^{n} S_i = \sum_{i=1}^{n} k_i \cdot \hat{\sigma}_{L_i} \cdot v_i \quad , \tag{2.17}$$

where:

S_i : safety stock in guilders for item i;

k_i : safety factor for item i;

$\hat{\sigma}_{L_i}$: estimated standard deviation of errors of forecasts over a replenishment lead time, in units.

Most practioners will find it difficult to explicitly decompose the total inventory budget into the above mentioned components. Second, such an explicit decomposition of the total budget will lead to inferior results compared to the simultaneous approach elaborated in the next section.

Finally we should note that demand is considered known and more or less constant in time. However, with stochastic demand things are more complicated. To treat these complexities appropriately we will introduce a three-dimensional aggregate analysis in the next section.

3. Three-dimensional aggregate inventory analysis: a modification of the Gardner and Dannenbring approach

Stochastic demand complicates matters considerably. First, cycle and safety stock investment decisions are interdependent for each item. In addition interactions also exist between the products as the two budgets for cycle and safety stocks have to be allocated to the individual items.
Until now we have assumed a predetermined total safety stock budget (see formula (2.17)). Furthermore the order quantities were calculated neglecting the standard deviation of forecast errors, as demand was deterministic and constant in time. With stochastic demand the standard deviations play an essential role in the determination of safety stocks.
Therefore the concept as presented in section 2, has to be extended with another aggregate policy variable: customer service in terms of the percentage of annual customer requisitions which are backordered (short), which will be projected on the vertical axis, denoted as $(1-P_2)$.
To allow for a simultaneous approach we will use an extended investment concept. Instead of total average cycle stock (y) we now introduce total stock investment (Y).
Just as in the two-dimensional analysis we will now present a graphical illustration of the relationships between total stock investment, workload and customer service.

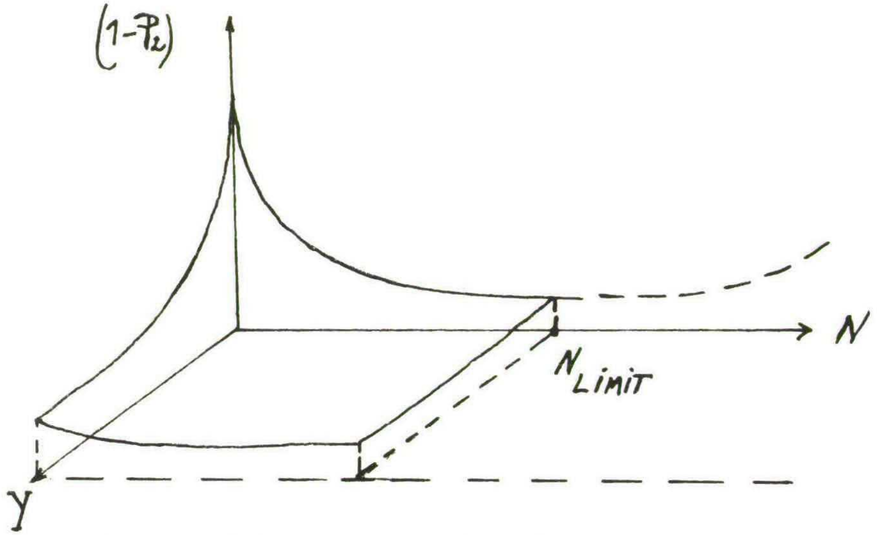

Figure 3.1. The optimal policy surface for stochastic demand

Most of the tradeoffs which are presented by the optimal policy surface are straightforward. The interested reader is referred to Gardner and Dannenbring [4] for more details. However, the effect of an increase in workload is more complex. In Gardner and Dannenbring's words: "With a fixed investment constraint, increases in workload are equivalent to increases in the number of exposures to risk of stockout".
On the other hand, the increased workload leads to increased total safety stock at the cost of total cycle stock.
For small and moderate values of N the marginal increase of safety stock dominates the risk of going short. But eventually the decrease in service level will overwhelm the marginal increase in safety stock. We define N_{limit}- for a fixed value of Y- as the value of N where a maximum service level is attained.
For determining the N_{limit}- value, Gardner and Dannenbring [4] formulate a Lagrangean model which minimizes the total number of annual shortages subject only to an investment constraint. Next they add a workload constraint to locate interior points on the optimal policy surface.
For more details on this subject we refer to their article [4].

We will now present an alternative approach which we think will be far more appealing to management. Instead of minimizing the service level in terms of the percentage of annual customer requisitions which are backordered (short) subject to a fixed workload constraint, we now try to minimize the total workload subject to a predetermined aggregate service constraint.

Before turning to the essential modified Lagrangean model formulation, we first present the overall aggregate simultaneous solution procedure.

Phase 1:

Locate the edge of optimality. In other words: determine the maximal workload N_{limit} given the investment constraint. Following Gardner and Dannenbring a single point on the edge of optimality is found by minimizing the following objective function:

$$\sum_{i=1}^{n} \frac{d_i}{q_i} \int_{s_i}^{\infty} \frac{(x_i - s_i)}{m_i} f(x_i)\, dx_i = (1-P_2) \tag{3.1}$$

subject to the investment constraint:

$$\sum_{i=1}^{n} \left[\frac{q_i \cdot v_i}{2} + S_i\right] = Y \quad , \tag{3.2}$$

where the symbols used will be defined at the end of phase 3. For details on the solution procedure we refer to Gardner and Dannenbring [4].

Phase 2:

Via a trial-and-error procedure one has to determine a feasible $(Y,(1-P_2))$ combination, where $(1-P_2)$ may vary. Following Gardner and Dannenbring an interior point associated with the $(Y,(1-P_2))$ combination is found by adding to (3.1) and (3.2) the workload constraint N_{limit} as was determined in Phase 1:

$$\sum_{i=1}^{n} \frac{d_i}{q_i} = N_{limit} \tag{3.3}$$

Again we refer to Gardner and Dannenbring [4], for further details on the solution procedure of this model.

Phase 3:

Determine the interior point on the optimal policy surface, according to the modified Lagrange model formulation. To locate any interior point on the surface, to the left of the edge of optimality, the objective function is:

$$\min_{q_i} z = \sum_{i=1}^{n} \frac{d_i}{q_i} \tag{3.3}$$

subject to the investment and service constraints:

$$\sum_{i=1}^{n} \left[\frac{q_i v_i}{2} + S_i\right] = Y \tag{3.4}$$

$$\sum_{i=1}^{n} \frac{d_i}{q_i} \int_{s_i}^{\infty} \frac{(x_i - s_i)}{m_i} f(x_i)\, dx_i = (1-P_2) \quad , \tag{3.5}$$

where

- z : total annual replenishments per year
- d_i : annual sales in units for item i
- q_i : order quantity in units for item i
- v_i : unit value of item i expressed in guilders per unit
- S_i : safety stock in guilders per item i
- Y : total investment constraint in guilders
- x_i : leadtime demand in units for item i
- m_i : customer requisition size in units for item i
- $f(x_i)$: probability density function for leadtime demand for item i
- $(1-P_2)$: service constraint in terms of the percentage of annual customer requisitions which are backordered (short).
- s_i : reorder points in units (sum of safety stock plus leadtime demand stock) for item i

The next step is to form the Lagrangean function, L:

$$L\,(q_1,\dots q_n, S_1,\dots S_n, \lambda_Y, \lambda_p) = \sum_{i=1}^{n} \frac{d_i}{q_i} + \lambda_Y \left[\sum_{i=1}^{n} \left[\frac{q_i v_i}{2} + S_i\right] - Y\right] +$$

$$+ \lambda_p \left[\sum_{i=1}^{n} \frac{d_i}{q_i m_i} \int_{s_i}^{\infty} (x_i - s_i)\, f(x_i)\, dx_i - (1-P_2)\right] \qquad (3.6)$$

Differentiating with respect to q_i, S_i, λ_Y and λ_p, we obtain the first order conditions:

$$\frac{\partial L}{\partial q_i} = \frac{-d_i}{q_i^2} + \frac{\lambda_p \cdot v_i}{2} - \frac{\lambda_p \cdot d_i}{q_i^2 \cdot m_i} \cdot \int_{s_i}^{\infty} (x_i - s_i)\, f(x_i)\, dx_i = 0$$

$$, \; (i=1,\dots n,) \qquad (3.7)$$

$$\frac{\partial L}{\partial S_i} = \lambda_Y + \frac{\lambda_Y \cdot d_i}{q_i \cdot m_i} \cdot \int_{s_i}^{\infty} (-1) \cdot f(x_i) \cdot dx_i = 0, \; (i=1,\dots,n) \qquad (3.8)$$

$$\frac{\partial L}{\partial \lambda_Y} = \sum_{i=1}^{n} \left[\frac{q_i \cdot v_i}{2} + S_i\right] - Y = 0 \qquad (3.9)$$

$$\frac{\partial l}{\partial \lambda_p} = \sum_{i=1}^{n} \frac{d_i}{q_i m_i} \int_{s_i}^{\infty} (x_i - s_i)\, f(x_i)\, dx_i - (1-P_2) = 0 \qquad (3.10)$$

We introduce some simplifying notation:

$$P_i = \int_{s_i}^{\infty} f(x_i)\, dx_i \qquad , \; (i=1,\dots,n) \qquad (3.11)$$

$$E_i = \int_{s_i}^{\infty} (x_i - s_i)\, f(x_i)\, dx_i \qquad , \; (i=1,\dots,n) \qquad (3.12)$$

$$F_i = \frac{d_i}{m_i} \qquad , \; (i=1,\dots,n) \qquad (3.13)$$

where

P_i: probability of a stockout during one order cycle

E_i: partial expectation of demand or the expected number of units short per order cycle

F_i: annual frequency of demand for each item.

So (3.7), (3.8), (3,9) and (3.10) are respectively equivalent with:

$$\frac{-d_i}{q_i^2} + \frac{\lambda_Y.v_i}{2} - \frac{\lambda_p.F_i.E_i}{q_i^2} = 0 \quad , (i=1,\ldots,n) \tag{3.14}$$

$$\lambda_Y - \frac{\lambda_p.F_i.P_i}{q_i} = 0 \quad , (i=1,\ldots,n) \tag{3.15}$$

$$\sum_{i=1}^{n} \left[\frac{q_i v_i}{2} + S_i\right] = Y \tag{3.16}$$

$$\sum_{i=1}^{n} \frac{F_i E_i}{q_i} = (1-P_2). \tag{3.17}$$

From (3.15) it follows that:

$$\lambda_Y = \frac{\lambda_p.F_i.P_i}{q_i} \text{ or } q_i = \frac{\lambda_p.F_i.P_i}{\lambda_Y}. \tag{3.18}$$

Relation (3.16) can be rewritten as follows:

$$\sum_{i=1}^{n} \left[\frac{\lambda_p.F_i.P_i.v_i}{2\lambda_Y} + S_i\right] = Y \text{ or } \lambda_Y = \frac{\lambda_p.\sum_{i=1}^{n} F_i.P_i.v_i}{2(Y-\sum_{i=1}^{n} S_i)}. \tag{3.19}$$

In addition, it follows from (3.15) that:

$$P_i = \frac{\lambda_Y.q_i}{\lambda_p.F_i} , (i=1,\ldots,n). \tag{3.20}$$

Rearrange formula (3.14) into:

$$\frac{\lambda_Y . v_i}{2} = \frac{d_i}{q_i^2} + \frac{\lambda_p . F_i . E_i}{q_i^2} \quad , \ (i=1,\dots,n) \tag{3.21}$$

or even further into:

$$\frac{\lambda_Y . q_i . v_i}{2} = \frac{d_i}{q_i} + \frac{\lambda_p . F_i . E_i}{q_i} \quad , \ (i=1,\dots,n). \tag{3.22}$$

Summing formula (3.22) with respect to i:

$$\lambda_Y \sum_{i=1}^{n} \frac{q_i . v_i}{2} = \sum_{i=1}^{n} \frac{d_i}{q_i} + \lambda_p \ . \ \sum_{i=1}^{n} \frac{F_i E_i}{q_i}, \tag{3.23}$$

which can be rearranged into:

$$\lambda_p = \frac{\lambda_Y . \left[\sum_{i=1}^{n} \frac{q_i v_i}{2}\right] - \left[\sum_{i=1}^{n} \frac{d_i}{q_i}\right]}{\sum_{i=1}^{n} \left[\frac{F_i E_i}{q_i}\right]}. \tag{3.24}$$

Substitution of (3.17) into (3.24) leads to:

$$\lambda_p = \frac{\lambda_Y \left[\sum_{i=1}^{n} \frac{q_i v_i}{2}\right] - \left[\sum_{i=1}^{n} \frac{d_i}{q_i}\right]}{(1-P_2)}. \tag{3.25}$$

We summarize the derived relationships:

$$\frac{\lambda_Y}{\lambda_p} = \frac{\sum_{i=1}^{n} F_i . P_i . v_i}{2 \ (Y - \sum_{i=1}^{n} S_i)} \tag{3.26}$$

$$P_i = \frac{\lambda_Y}{\lambda_p} \ . \ \frac{q_i}{F_i} \tag{3.20}$$

$$\lambda_p = \frac{\lambda_Y \left[\sum_{i=1}^{n} \frac{q_i . v_i}{2}\right] - \left[\sum_{i=1}^{n} \frac{d_i}{q_i}\right]}{(1-P_2)}. \tag{3.25}$$

Finally we derive an equation for q_i, (i=1,...,n) from equation (3.21):

$$q_i^2 = \frac{2\ (d_i+\lambda_p.F_i.E_i)}{\lambda_Y.v_i} \quad , \ (i=1,\dots,n) \qquad (3.27)$$

which can be reduced to:

$$q_i = \sqrt{\frac{2\ (d_i+\lambda_p.F_i.E_i)}{\lambda_Y.v_i}} \quad , \ (i=1,\dots,n). \qquad (3.28)$$

A detailed flow-chart of the solution procedure can be found in the appendix, where the following assumptions are in order:
- the lenght of the leadtime is constant;
- the customer requisition sizes for each items are constants and are independent of the level of demand;
- the leadtime demand is normally distributed, where the distribution moments are given and may differ per item.

Brown [2], Pantumsinchai, Hassan and Gupta [7] among others have also proposed aggregate inventory decision rules, for a stochastic demand process. However, their suggested procedures are inferior to the one above, because:
- reorder quantities and reorder points are determined sequentially, i.e. not simultaneously;
- reorder quantities and reorder points are determined neglecting any influences of standard deviations of forecast errors or service levels;
- total average cycle stock investment is considered as a resultant, as total safety stock investment is determined first and independent of total average cycle stock investment.

4. Conclusions and suggestions for further research

As mentioned earlier, aggregate concepts are more appealing to management than any analysis on individual basis.
In addition we avoid all estimation problems of an explicit marginal cost determination by working with aggregate variables.

The service measure as used in this paper is based on an average value for a group of items under consideration. A measure which takes into account the same service level for every item in the group could be an alternative. This concept would imply the use of n service constraints which can be attacked with existing software packages on non-linear programming. However, the computational effort is more complicated. In our searchprocedure optimal reorder quantities and safety stocks are determined simultaneously. The standard deviations of forecasts errors during the leadtime may now influence the optimal results. In this way items which can be forecasted accurately consume less safety stock than those which are highly erratic in nature.

Furthermore, in comparison with the sequential two-dimensional analysis, where safety stocks are predetermined, we now obtain a more efficient balance between total average cycle stock - and total safety stock investment.

Gardner and Dannenbring (see [4]) have already compared a simultaneous reorder procedure with one where reorder quantities were determined independently from each other, and they obtained substantial improvements. Further research is planned to test our suggested procedure empirically.

References:

[1] Abramowitz, M. and Stegun, J.A., 1968,
Handbook of mathematical functions,
Dover Publications, New York.

[2] Brown, R.G., 1967,
Decision rules for inventory management,
Arthur D. Little, New York.

[3] Eaton, J.A., 1964,
New - the limit technique,
Modern Materials Handling, 19: 38-43.

[4] Gardner, E.S. and Dannenbring, D.G., 1979,
Using optimal policy surfaces to analyze aggregate inventory tradeoffs,
Management Science, 25: 709-720.

[5] Groff, G.K. and Muth, J.F., 1972,
Operations management: analysis for decisions,
Irwin, Homewood.

[6] Hadley, G. and Whitin, T.M., 1963,
Analysis of inventory systems,
Prentice-Hall, Englewood Cliffs.

[7] Pantumsinchai, P., Hassan, M.Z. and Gupta, I.D., 1983,
Basic programs for production and operations management,
Prentice-Hall, Englewood Cliffs.

[8] Plossl, G.W. and Wight, O.W., 1967
Production and inventory control,
Prentice-Hall, Englewood Cliffs.

[9] Prichard, J.W. and Eagle, R.H., 1965,
Modern inventory management,
New York.

[10] Schonberger, R.J. and Schniederjans, M.J., 1984,
Re-inventing inventory control,
Interfaces, 14: 76-83.

[11] Selen, W.J. and Wood, W.R., 1987,
Inventory cost definition in an EOQ model application,
Production and Inventory Management Journal, 44-47.

[12] Silver, E.A. and Peterson, R., 1985,
Decision systems for inventory management and production planning,
John Wiley & Sons, New York.

Appendix: Flowchart of the solution procedure

START

Reading of all starting values:

Y; n; $\hat{x}_{L_i}$; $\hat{\sigma}_{L_i}$; d_i; v_i; m_i; $i=1,\ldots,n$

Phase 1:

Having determined N_{limit}, using the solution procedure of Gardner and Dannenbring, we will only use workload values which are smaller than N_{limit}.

Phase 2:

We check the feasibility of the service level $(1-P_2)^*$ as fixed by top management, by determining the minimal service level in terms of the percentage of annual customer requisitions which are backordered (short), $(1-P_2)$ associated with the (Y,N_{limit}) combination from Phase 1

$(1-P_2) > (1-P_2)^*$

Yes

No

Determine $\frac{\lambda_y}{\lambda_p}$ using (3.26):

$$\frac{\lambda_Y}{\lambda_p} = \frac{\sum_{i=1}^{n} F_i \; P_i \; v_i}{2\,(Y - \sum_{i=n}^{n} S_i)} \qquad \text{where } S_i = 0, \quad P_i = \tfrac{1}{2},\ i=1,\dots,n$$

$$\text{So: } \frac{\lambda_Y}{\lambda_p} = \frac{\sum_{i=1}^{n} \tfrac{1}{2} F_i \cdot v_i}{2Y}$$

Calculate q_i, $i=1,\dots,n$ by means of formula (3.20)

$$q_i = \frac{\lambda_p \cdot F_i \cdot P_i}{\lambda_Y} \qquad \text{where } P_i = \tfrac{1}{2},\ i=1,\dots,n,$$

$$\text{so } q_i = \frac{\tfrac{1}{2}\,\lambda_p \cdot F_i}{\lambda_Y}$$

Determine λ_Y and λ_p with the help of equation (3.25)

$$\lambda_p = \frac{\lambda_Y \left(\sum_{i=1}^{n} \frac{q_i v_i}{2}\right) - \sum_{i=1}^{n} \frac{d_i}{q_i}}{(1-P_2)},$$

Calculate q_i = i=1,...,n using (3.28)

$$q_i = \sqrt{\frac{2.(d_i+\lambda_p.F_i.E_i)}{\lambda_Y.v_i}},\ i=1,\ldots,n$$

where:

$$E_i = \int_{s_i}^{\infty} (x_i-s_i)\ f(x_i)\ dx_i$$

$$\approx \hat{\sigma}_{\bar{L}_i} \left\{\frac{1}{\sqrt{2\pi}} \exp\ (-\tfrac{1}{2}\ k_i^2) - k_i \frac{1}{\sqrt{2\pi}} \exp\ (-\tfrac{1}{2}\ k_i^2)\ [b_1 t_i + \ldots + b_5 t_i^{\,5}]\right.$$

and: $t_i = \dfrac{1}{1+0{,}2316419.k_i}$, i=1,...,n

$b_1 = 0{,}319381530$

$b_2 = -0{,}356563782$

$b_3 = 1{,}781477937$

$b_4 = -1{,}821255978$

$b_5 = 1{,}3302784429$

Evaluate P_i, i=1,...,n with the help of (3.20)

$$P_i = \frac{\lambda_Y}{\lambda_p} \frac{q_i}{F_i},\ i=1,\ldots,n$$

Now P_i, i=1,...,n is known where also

$$P_i = \int_{S_i}^{\infty} f(x)\, dx_i = \int_{k_i}^{\infty} f_x(z)\, dz \text{ with } z \sim N(0,1)$$

Using a rational function approzimation (see Abramowitz and Stegun [1]):

$$k_i \approx t - \frac{c_0 + c_1 . t + c_2 . t^2}{1 + d_1 . t + d_2 . t^2 + d_3 . t^3} \quad \text{where}$$

$$t = \sqrt{\ln \left[\frac{1}{P_i^2}\right]} \quad \text{with } P_i \text{ known, } i=1,\dots,n$$

where $c_0 = 2{,}515517$ $d_1 = 1{,}432788$

$c_1 = 0{,}802853$ $d_2 = 0{,}189269$

$c_2 = 0{,}010328$ $d_3 = 0{,}001308$

Calculate the S_i values, i=1,...,n associated with the current k_i values, i=1,...,n:

$$S_i = k_i \,.\, \hat{\sigma}_{\bar{L}_i} \,.\, v_i \text{ with } \hat{\sigma}_{\bar{L}_i},\ v_i \text{ known}$$

Calculate:

$$\sum_{i=1}^{n} \left[\frac{q_i v_i}{2} + S_i\right] \text{ and } \sum_{i=1}^{n} \frac{F_i}{q_i} \,.\, E_i$$

$$\sum_{i=1}^{n} \left[\frac{q_i v_i}{2} + S_i\right] \stackrel{?}{=} Y$$

and

$$\sum_{i=1}^{n} \frac{F_i}{q_i} \cdot E_i \stackrel{?}{=} (1-P_2)^*$$

STOP AND PRINT ALL THE RELEVANT INFORMATION INCLUDING $\sum_{i=1}^{n} \frac{d_i}{q_i} = N$ AND S_i, $i=1,\ldots,n$.

WITH FORMULA (2.15) WE CAN CALCULATE THE INDIVIDUAL EOQ_i, $i=1,\ldots,n$ WHERE

$$y = Y - \sum_{i=1}^{n} S_i$$

NOTE THE DIFFERENCES BETWEEN q_i AND EOQ_i, $i=1,\ldots,n$.

No

Recalculate $\frac{\lambda_Y}{\lambda_p}$ using (3.26):

$$\frac{\lambda_Y}{\lambda_p} = \frac{\sum_{i=1}^{n} F_i \cdot P_i \cdot v_i}{2 \cdot (Y - \sum_{i=1}^{n} S_i)}$$

Recalculate λ_Y and λ_p with the help of (3.25)

$$\lambda_p = \frac{\lambda_Y \cdot \left[\sum_{i=1}^{n} \frac{q_i v_i}{2}\right] - \sum_{i=1}^{n} \frac{d_i}{q_i}}{(1-P_2)}$$

where $\frac{\lambda_Y}{\lambda_p}$, q_i, v_i, d_i, $(1-P_2)$ are known

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